Reading 66: Introduction to the Measurement of Interest Rate Risk

LOS 66a: Distinguish between the full valuation approach (the scenario analysis approach) and the duration/convexity approach for measuring interest rate risk, and explain the advantage of using the full valuation approach. Vol 5, pg 444-447

We know from earlier readings that the major risk faced by bond investors is the risk that interest rates will rise and result in a decrease in the value of their investments. There are two ways to measure this risk.

The full valuation or scenario analysis approach *actually calculates* the change in value of a fixed income security or portfolio in response to a change in interest rates. It calculates the change in value of bonds under various scenarios including:

- A parallel shift in the yield curve i.e., YTMs for all maturities change by the same amount. An example of a parallel shift in the yield curve is when yield across all maturities move up by 50 basis points.
- A change in the slope or shape of the yield curve i.e., non-parallel movements in the yield curve. An example of a change in the slope of the yield curve is when short term yields rise by 100 basis points and long term yields only rise by 50 basis points.

Example 1: The Full Valuation Approach of Measuring Interest Rate Risk

Consider the following option-free bonds:

Bond A is a 10% annual-pay bond with 5 years to maturity, and is priced at $107.985 to yield 8%.

Bond B is a 7% annual-pay bond with a maturity of 12 years, and is priced at $88.98 to yield 8.5%.

Calculate the change in value of a position worth $1 million of par in each of the bonds under the following scenarios:

1. A 50 basis point upward parallel shift in the yield curve.
2. A 100 basis point upward parallel shift in the yield curve.

Solution

Calculation of Bond A's prices:

Scenario 1:
N = 5; PMT = -$100,000; FV = -$1,000,000; I/Y = 8% + 0.5%; CPT PV; PV → $1,059,110

Scenario 2:
N = 5; PMT = -$100,000; FV = -$1,000,000; I/Y = 8% + 1%; CPT PV; PV → $1,038,897

Calculation of Bond B's prices:

Scenario 1:
N = 12; PMT = -$70,000; FV = -$1,000,000; I/Y = 8.5% + 0.5%; CPT PV; PV → $856,785

Scenario 2:
N = 12; PMT = -$70,000; FV = -$1,000,000; I/Y = 8.5% + 1%; CPT PV; PV → $825,404

The following table lists the prices that we have calculated:

Scenario	Change in Yields	Value of Bond A	Value of Bond B	Change in Price of Bond A	Change in Price of Bond B
Current		$1,079,850	$889,800		
1	+50 bp	$1,059,110	$856,785	-1.92%	-3.71%
2	+100 bp	$1,038,897	$825,404	-3.79%	-7.24%

The change in price of Bond B is more significant that change in price of Bond A. This is understandable because Bond B has a *longer* term to maturity and a *lower* coupon rate, which imply greater interest rate risk (duration).

In earlier readings we determined that there is an *inverse, convex* relationship between interest rates and bond prices. An *increase* in yields leads to a *decrease* in bond prices, while a *decrease* in yields leads to an *increase* in prices. Also recall the following properties of fixed income securities.

- *Higher* coupon means *lower* interest rate risk (duration). *Lower* coupon means *higher* interest rate risk.
- *Longer* maturity means *higher* interest rate risk.
- *Higher* current market yields mean *lower* interest rate risk.

For more complex bonds (e.g. those with embedded options) simulation models that incorporate interest rate volatility and yield changes are used to measure the interest rate sensitivity of bond prices. *Stress-testing* is the practice of using extreme scenarios and evaluating their effect on bond values. The full valuation approach can be used to assess the exposure of a fixed income security or portfolio to interest rate changes under any given scenario as long as the assumptions made under the valuation model hold. Applied to a portfolio of bonds one at a time, the full valuation approach offers a very accurate measure of how different yields and interest rate volatility scenarios affect the value of the portfolio. While this approach is the theoretically preferred approach, it is very time consuming and arduous in nature.

The duration/convexity approach is a simpler, two-step approach to evaluate the interest rate sensitivity of a fixed income security or portfolio. Its main advantage is its simplicity compared to the full valuation approach.

Los 66b: Demonstrate the price volatility characteristics for option-free, callable, prepayable, and putable bonds when interest rates change. Vol 5, pg 448-456

Los 66c: Describe positive convexity, negative convexity, and their relation to bond price and yield. Vol 5, pg 448-456

Let's work with the following example to understand why the *convex* shape of the price-yield relationship of fixed income securities is so important:

A 5-year 10% annual-coupon bond is currently selling for par ($100). This implies that its current yield to maturity is 10% (same as the coupon rate). Let's calculate the change in the bond's price under four scenarios:

a. Yields move up by 25bp.
b. Yields move down by 25bp.
c. Yields move up by 150bp.
d. Yields move down by 150bp.

Calculations:

a. Yields move up by 25bp

N = 5; PMT = -$10; FV = -$100; I/Y = 10.25; CPT PV; PV → $99.06

Percentage price change = -0.94%

b. Yields move down by 25bp

N = 5; PMT = -$10; FV = -$100; I/Y = 9.75; CPT PV; PV → $100.95

Percentage price change = 0.95%

c. Yields move up by 150bp

N = 5; PMT = -$10; FV = -$100; I/Y = 11.5; CPT PV; PV → $94.53

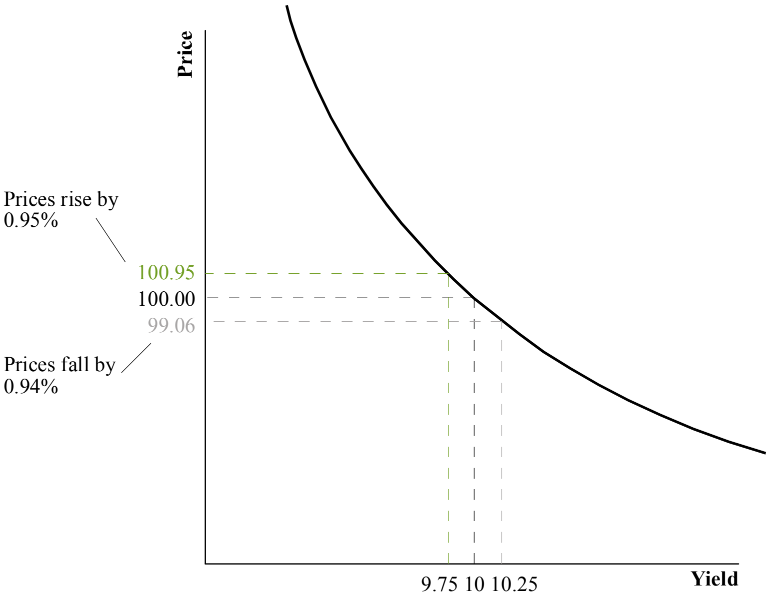
Percentage price change = -5.47%

d. Yields move down by 150bp

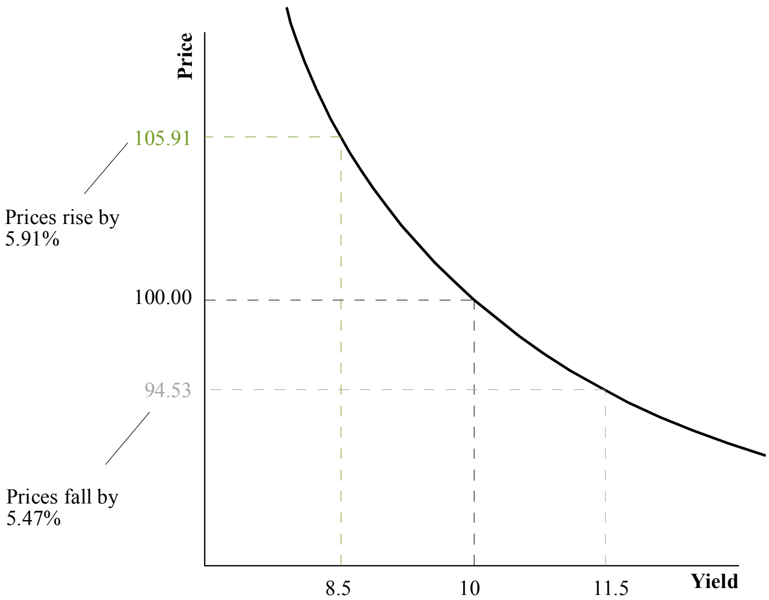
N = 5; PMT = -$10; FV = -$100; I/Y = 8.5; CPT PV; PV → $105.91

Percentage price change = 5.91%

Figure 1: Change in Price of an Option Free Bond



1a. Change in price when yields rise by 25 bps



1b. Change in price when yields rise by 150 bps

The above calculations and graphs illustrate the following important characteristics of the price-yield relationship of option-free bonds:

- For *small* changes in yields (25 bp), the percentage price change for a given bond is *roughly the same* regardless of whether yields move up or down (-0.94% and 0.95%).

- For larger changes in yields (150 bp), the percentage price *increase* resulting from a decrease in yields (5.91%) is *greater* than the percentage price *decrease* from an increase in yields (-5.47%).

The implication of all this is that an investor in an option-free bond has *more* to gain from a decrease in yields, than she has to lose from an identical increase in yields. Due to the convex shape of the price-yield relationship, bond prices fall at a *decreasing* rate as yields *increase*, and increase at an *increasing* rate as yields *decrease*.

Duration simply measures the slope of the price-yield profile at a particular yield level. For small changes in yields, duration accurately estimates the change in bond prices. However, with larger change in yields, the convexity of the price-yield profile results in duration-based price estimates being relatively inaccurate. To estimate bond prices more accurately a convexity adjustment is required. We will study the convexity adjustment later in the reading.

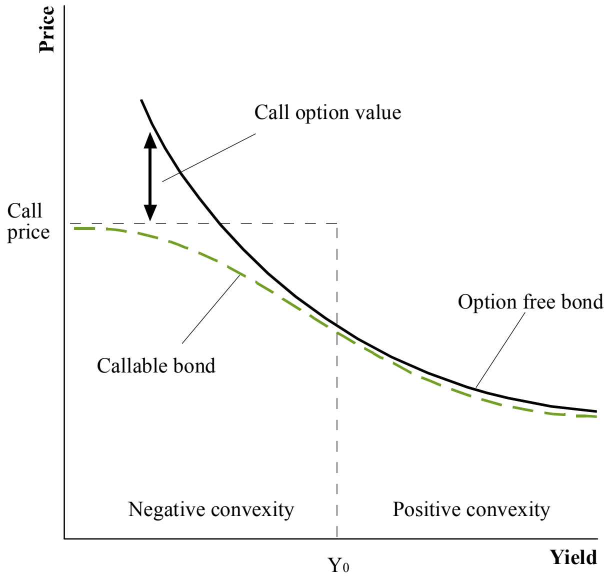
Bonds With Call and Prepayment Options

Callable bonds grant the issuer the option to redeem bonds prior to maturity. When interest rates fall below the coupon rate, callable bond issuers call their bonds at the predetermined call prices and realize economic benefits by retiring debt obligations for lower than their market value. Effectively, the price upside for callable bond investors is capped at the call price.

Prices of straight bonds rise more significantly in response to a given decrease in yields than they fall from an identical increase in yields. This feature is known as *positive convexity* and works to the benefit of investors in fixed income securities. However, for callable bonds the price yield relationship exhibits positive convexity *only at higher yields*. At lower yields, callable bonds exhibit *negative convexity* i.e., for a given change in yields, the price appreciation is *less* than the price decline. At a certain yield level, there is very little appreciation in the price of callable bonds as yields decline further (because it becomes increasingly likely that they will be called), and the bond exhibits ‘price compression’.

Figure 2: Callable Bond

At low yields the embedded call option prevents the callable bond's price from rising beyond the call price.



At high interest rates the price yield profile of a callable bond is similar to that of a straight bond.

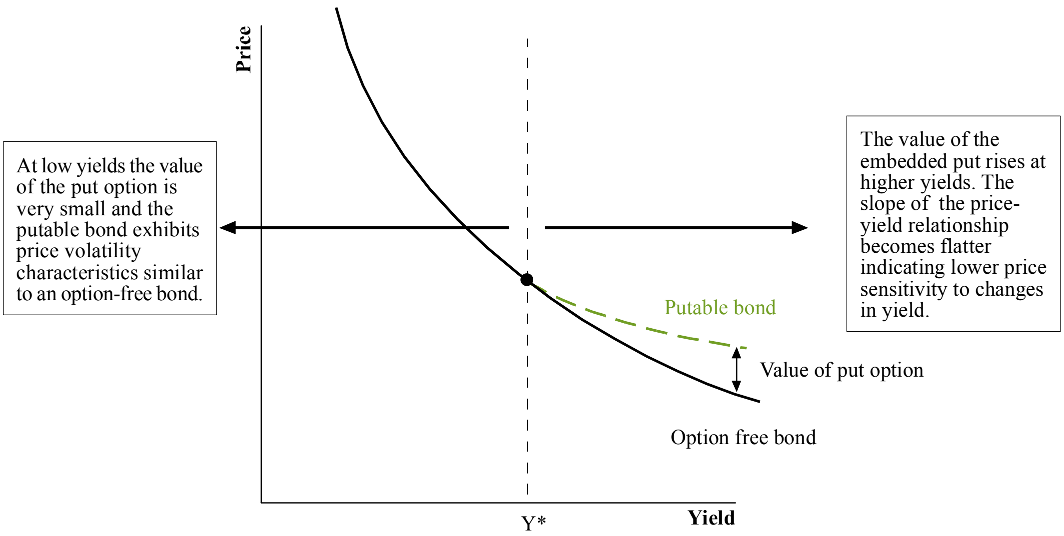
The effect of a prepayment option is similar to that of the call option in a callable bond- it results in negative convexity at low yields and reduces the price volatility of the security. Although callable and prepayable securities exhibit low interest rate risk at lower yields, they suffer from greater reinvestment risk. At lower yields the probability that the issuer will call the bond, or that a mortgage issuer (homeowner/borrower) will prepay her obligations increases, and consequently magnifies the risk that an investor might have to reinvest the proceeds at the newer, lower interest rates.

Bonds with Embedded Put Options

Putable bonds can be sold back to the issuer on specific dates at predetermined put prices. Typically, the put price equals the bond's par value. In the case of putable bonds, protection is offered to investors. If yields rise, straight bond prices fall, but an option to put the bond back to the issuer will hold value for an investor because it offers her price protection on the downside.

At low yield levels, the price of the putable bond moves very similarly to the price of a straight bond in response to changes in interest rates. As rates rise, the embedded put option increases in value, and cushions the price reduction.

Figure 3: Putable Bonds



LOS 66d: Compute and interpret the effective duration of a bond, given information about how the bond's price will increase and decrease for given changes in interest rates, and compute the approximate percentage price change for a bond given the bond's effective duration and a specified change in yield. Vol 5, pg 456-463

Duration is a *linear, approximate* measure of the changes in the price of a bond in response to yield changes. More specifically, it is approximate change in the price of a bond when yields change by 100 basis points.

We will mainly be working with effective duration in calculations involving duration. The formula for effective duration uses the difference between two prices (the price when yields increase and decrease by a given amount) in the numerator.

If we are working with bonds with embedded options, the prices used are adjusted to reflect both, changes in yields and changes in expected cash flows because of the option features of the bond.

$$\text{Duration} = \frac{V_{-} - V_{+}}{2(V_0)(\Delta y)}$$

where:
Δy = change in yield in decimal
V_0 = initial price
V_{-} = price if yields decline by Δy
V_{+} = price if yields increase by Δy

Example 2: Calculating Effective Duration

A 10-year annual-pay bond with a 9% coupon rate is currently priced at $938.55 to yield 10%. If yields decline by 25 basis points to 9.75%, the bond's price will increase to $953.42 and if yields increase by 25 basis points to 10.25%, its price will decrease to $924.01. Based on these prices and yield changes, calculate the effective duration of this bond

Solution

$$\text{Effective duration} = \frac{(\text{bond price when yields fall - bond price when yield rise})}{2 * (\text{initial price}) * (\text{Change in yields in decimals})}$$

$$\text{Effective duration} = \frac{V_{-} - V_{+}}{2\, V_0\, (\triangle y)}$$

$$\text{Effective duration} = \frac{(\$953.42 - \$924.01)}{2 * \$938.55 * 0.0025} = 6.27$$

Interpretation: A bond with a duration of 6.27 would witness a change in value of approximately 6.27% when there is a 100 basis point or 1% parallel shift in the yield curve.

One source of confusion among many candidates is the disconnect between the change in yields used to calculate BV_{+} and BV_{-} (25 basis points in Example 2) and the 100 basis points in the interpretation of duration. The answer lies in the manner that the formula for effective duration is structured. While the conclusion that any duration figure calculated by the above formula is for a 100 basis points change in yields regardless of the change in yields used to calculate bond prices in the numerator, can be derived mathematically, we think it serves our interests here better to just accept that this is the case.

Conclusion: No matter what change in yields is used to calculate bond prices in the numerator, the duration figure calculated from the formula above will always give us the percentage change in the price of a bond in response to a 100 basis point change in yields. To calculate the change in price of the bond when interest rates change by 50 basis points for example, would require halving the calculated duration value.

Example 3: Calculating the Percentage Price Change for a Bond.

Based on an effective duration of 6.27 calculate the approximate percentage price change for the bond in the previous example for a 75 basis point change in yields.

Solution

Approximate percentage price change = - duration * $\triangle y$ in percent

Approximate percentage price change = - 6.27 * 0.0075 * 100

Approximate percentage price change = -4.703%

In Table 1, we compare the duration-based price estimates of the bond in Example 2 to the actual price of the bond in response to a 10 basis point and a 100 basis point change in yields.

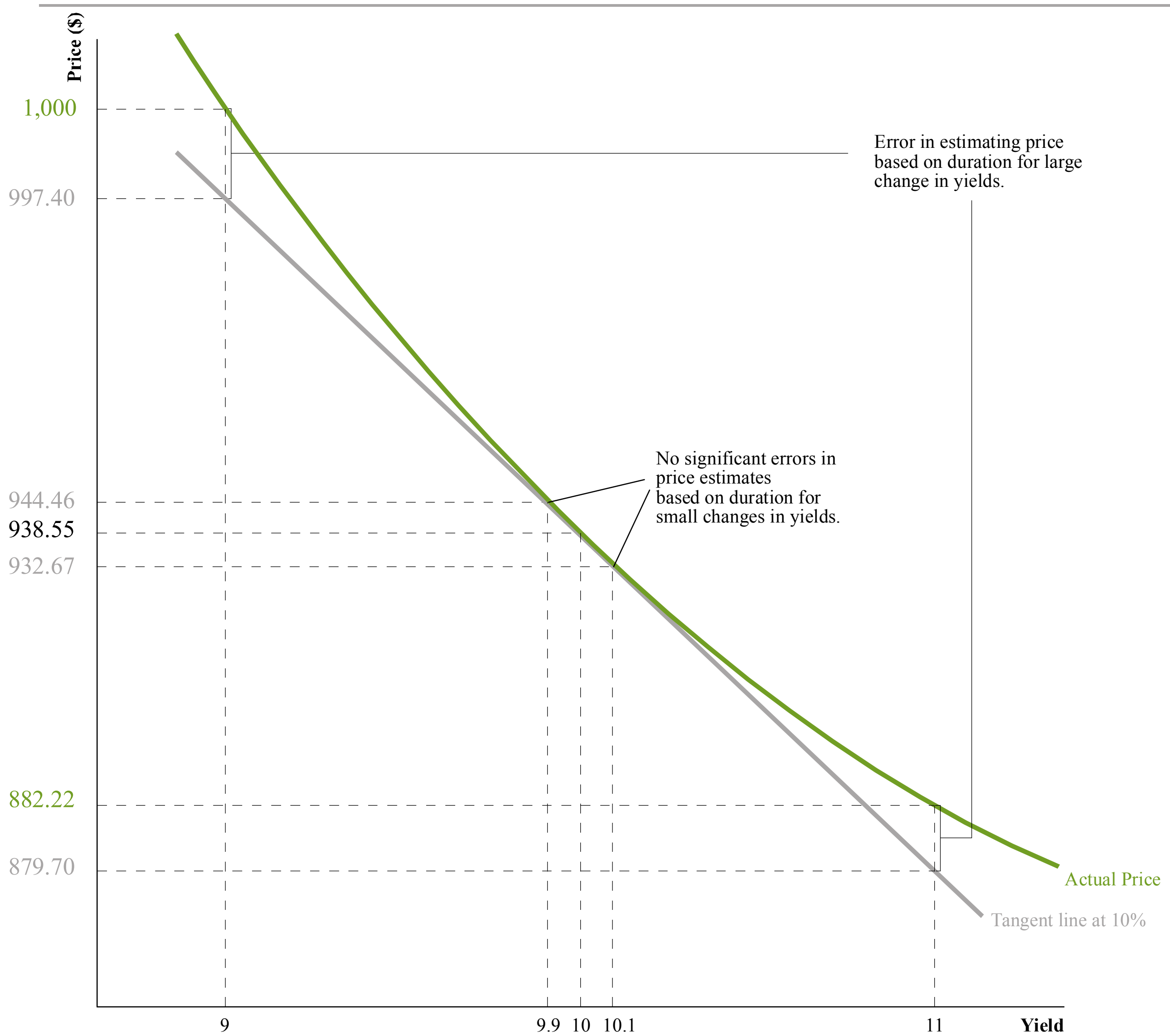
Table 1: Comparison of Price Estimates Based on Duration

		Price estimate		% Price change		
Yield Change (bp)	**Initial price**	**Based on duration**	**Actual**	**Based on duration**	**Actual**	**Comment**
+10	$938.55	$932.67	$932.70	-0.627%	-0.623%	Price estimate based on duration is close to the actual price.
-10	$938.55	$944.43	$944.46	+0.627%	+0.63%	Price estimate based on duration is close to the actual price.
+100	$938.55	$879.70	$882.22	-6.27%	-6.00%	Price estimate based on duration underestimates the actual price.
-100	$938.55	$997.40	$1,000	+6.27%	+6.55%	Price estimate based on duration underestimates the actual price.
		(1 - 0.0627)(938.55)	PMT = -$90; I/Y = 11; FV = -$1,000; N = 10; CPT PV			

Table 1 allows us to reemphasize a few important points that we have mentioned earlier:

- Duration is an accurate measure of changes in bond prices only in response to *small* changes in yield.
- The percentage price *increase* brought about by a given decrease in yields is *greater* than the percentage price *decrease* brought about by a given increase in yields.
- When yields change more significantly, using duration to estimate bond prices will *underestimate* the price increase, and *overestimate* the price decrease. In both cases, price estimates based on duration fall *short* of actual prices.

Figure 4: Duration Based Price Estimation



LOS 66e: Distinguish among the alternative definitions of duration, and explain why effective duration is the most appropriate measure of interest rate risk for bonds with embedded options. Vol 5, pg 464-465

Modified duration is an estimate of the percentage change in the price of a bond in response to a 100bp change in yields *assuming that the bond's cash flows do not change when yields change*. This assumption holds true only for option-free bonds.

Effective duration is an estimate of the percentage change in a bond's price in response to a 100bp change in yields *after accounting for any changes in the bond's expected cash flows due to yield changes*. Therefore, it is the more appropriate measure of interest rate risk for bonds with embedded options. For bonds with no embedded options, modified duration and effective duration can both be used.

Macaulay Duration is the weighted average maturity of a bond where the individual weights are based on the proportion of total discounted cash flows to be received in each period. You don't need to worry about its definition or calculation; just remember that it is a figure derived from modified duration. Because modified and Macaulay duration do not consider possible changes in a bond's cash flows, they are flawed measures of the price sensitivity of bonds with embedded options to changes in interest rates.

Interpretations of Duration

Duration can be interpreted in three ways:

- Duration is the first derivative of the price-yield function. Mathematically speaking, the first derivative of a curve at any point is the slope or gradient of the curve at the given point. Therefore, given the convex shape of the price-yield relationship, the duration at any given yield is simply the slope of the price-yield relationship at that particular yield level.

- Duration is the weighted average time it takes for all the bond's cash flows to be realized. Each cash flow is weighted by the proportion of the bond's total cash flows that it comprises. Consequently, duration can also be expressed in years.

- Duration is simply the approximate change in the value of a bond in response to a 100 basis point change in interest rates (a 1% change in yields). This interpretation of duration is the most preferred one.

LOS 66f: Compute the duration of a portfolio, given the duration of the bonds comprising the portfolio, and explain the limitations of portfolio duration.
Vol 5, pg 468-469

A portfolio's duration can be calculated as the weighted average of the durations of the individual bonds it consists of. Each bond's weight equals the proportion of the total portfolio's *market* value that it comprises.

$$\text{Portfolio duration} = w_1D_1 + w_2D_2 + \ldots + w_ND_N$$

where:
N = Number of bonds in portfolio.
D_i = Duration of Bond i.
w_i = Market value of Bond i divided by the market value of portfolio.

Example 4: Calculating Portfolio Duration

Suppose we have a portfolio containing only two securities - Bond X and Bond Y. The market value of Bond X is $7,500 and that of Bond Y is $5,000. The duration of Bond X is 7.25, while that of Bond Y is 5.0. Calculate the duration of portfolio.

Solution

Each bond's weight in the portfolio is calculated as:

$$w_X = \frac{\$7,500}{\$12,500} = 60\%$$

$$w_Y = \frac{\$5,000}{\$12,500} = 40\%$$

Therefore, portfolio duration equals
0.6 * 7.25 + 0.4 * 5 = 6.35

Limitations of Portfolio Duration

Duration assumes that yields for all maturities change by the same amount, i.e.there is a parallel shift in the yield curve. Portfolios of bonds are composed of a variety of bonds that may have different maturities, credit risks, and embedded options, and realistically speaking, there is no reason to expect yields on all these bonds to change by the same amount.

LOS 66g: Describe the convexity measure of a bond and estimate a bond's percentage price change, given the bond's duration and convexity and a specified change in interest rates. Vol 5, pg 469-471

Duration suggests that bond prices approximately change by the same amount whether interest rates increase or decrease by a given number of basis points. Earlier in the reading, we observed that duration is a good approximation for changes in bond prices only for *small* changes in yields. However, for *large* changes in yields, duration *underestimates* the price increase caused by a reduction in yields, and *overestimates* the decrease in prices when yields rise. Therefore, price estimates of option-free bonds based on duration must be revised *upwards* to bring them closer to their actual values. This revision is performed via the convexity adjustment:

In Figure 4, we have already illustrated the importance of the convexity adjustment for approximating bond prices for larger changes in yields. The convexity adjustment is necessary to correct the error in duration-based estimates of bond prices. Duration would only provide an accurate measure of the change in prices if the price-yield relationship were linear. The *more* the curvature or convexity of the price-yield relationship, the *more significant* the convexity adjustment becomes. A bond's convexity is calculated as

$$C = \frac{V_+ + V_- - 2V_0}{2V_0(\Delta y)^2}$$

We can get a more accurate value for the percentage change in bond prices in response to a change in yields by combing duration and convexity:

$$\text{Percentage change in bond price} = \text{duration effect} + \text{convexity adjustment} = \{[-\text{duration} * (\Delta y)] + [\text{convexity} * (\Delta y)^2]\} * 100$$

where:
Δy = Change in yields in decimals.

Example 5: Estimating Bond Price Changes with Duration and Convexity

Consider a 9% Treasury bond that currently trades at $938.55 with a YTM of 10%. Calculate the percentage change in the price of the bond in response to a 1% increase, and a 1% decrease in yields assuming that the bond has a duration of 6.27 and a convexity of 28.13. Notice that we are working with the same bond as in Example 2.

Solution

Duration effect = -6.27 * 0.01 * 100 = -6.27%.

Convexity adjustment = 28.13 * 0.01^2 * 100 = 0.2813%.

Total effect for a *decrease in yields of 1%* (from 10% to 9%)
= 6.27 + 0.2813 = 6.55%

Estimate of new price of bond
= (1 + 0.065513) * $938.55 = $1,000.04.

Combining the effect of duration and convexity to estimate bond price changes brings us closer to the actual price of $1,000. Our estimate of the bond's price based on duration alone was $997.40 (see Table 1).

Total effect for an *increase in yields of 1%* from (10% to 11%)
= - 6.26 + 0.2813 = -5.99%

Estimate of new price of bond
= (1 – 0.059887) * $938.55 = $882.34.

Once again, using the duration/convexity approach brings us closer to the actual bond price ($882.22). Based on duration alone, our estimate of the bond's price in response to a 1% increase in yields stood at $879.70 (see Table 1).

Convexity and Bonds with Embedded Options

Option-free bonds always have a convex price-yield relationship so the convexity adjustment is always positive. However, bonds with embedded options may exhibit negative convexity. For example, callable bonds exhibit negative convexity at low yields. As we illustrated in Figure 2, the percentage price increase from a decrease in yields is actually *lower* than the percentage price decrease from an increase in yields for callable bonds at low yields. In such cases, the duration effect and the convexity adjustment are both negative.

LOS 66h: Differentiate between modified convexity and effective convexity.

Vol 5, pg 472.

There are two measures of convexity- effective convexity and modified convexity. We have been working with effective convexity in our examples in this reading.

Modified convexity assumes that the cash flows of a bond do not change when interest rates change. Therefore, modified convexity cannot be used for bonds with embedded options. The calculated value for modified convexity is always positive regardless of whether the bond contains an embedded option.

Effective convexity does not assume that the cash flows of a bond will not change when there is a change in yields. The effective convexity adjustment is negative for bonds with embedded options.

LOS 66i: Compute the price value of a basis point (PVBP), and explain its relationship to duration. **Vol 5, pg 472-473**

The price value of a basis point (PVBP) is the dollar change in the price of a bond when yields change by 1 basis point or 0.01%. While we can calculate the actual change in the price of a bond in response to a 1 bp change in yields, it is practically easier to use duration to estimate the price value of a basis point through this formula:

Price value of a basis point = Duration * 0.0001 * bond value

Example 6: Calculating the Price Value of a Basis Point

A bond has a market value of $200,000 and duration of 7.56. What is its price value of a basis point?

Solution

Duration measures the change in a bond's price in response to a 1% or 100 basis point change in yields. The percentage change in a bond's price for a 1 bp change in yields is simply 0.01 times the duration of the bond:

Change in bond price = 0.01 * 7.56% = 0.0756%

The price value of a basis point is therefore the percentage change in price multiplied by the value of the bond.

PVBP = 0.0756% * $200,000 = $151.20