Reading 65: Yield Measures, Spot Rates and Forward Rates

LOS 65a: Explain the sources of return from investing in a bond. Vol 5, pg 390-391

An investor expects the following sources of return from a fixed income security:

- Periodic coupon payments.
- Capital gains if the proceeds from selling the bond exceed the amount initially invested.
- Income from reinvestment of interim cash flows (coupon and/or principal payments received prior to maturity).

LOS 65b: Compute and interpret the traditional yield measures for fixed-rate bonds and explain their limitations and assumptions. Vol 5, pg 391-393

Current Yield

The current yield is the ratio of the annual *dollar* coupon to the *current* market price.

> The current yield is based on the annual coupon amount so the current yields for semiannual and annual-pay bonds with the same coupon rate and price are identical.

$$\text{Current yield} = \frac{\text{Annual cash coupon}}{\text{Bond price}}$$

Example 1: Computing the Current Yield

Calculate the current yield of a 10-year, $1,000 par 8% semiannual-pay bond that is trading at $1,320.50.

Solution

Annual cash coupon = Par value * Coupon rate = $1,000 * 0.08 = $80

$$\text{Current yield} = \frac{80}{1{,}320.50} = 0.0606 \text{ or } 6.06\%$$

Remember the following relationships:

- The current yield is *greater* than the coupon rate when the bond is trading at a *discount*.
- The current yield is *lower* than the coupon rate when the bond is trading at a *premium*.

Limitation of the Current Yield

- Only considers coupon income and ignores other sources of investment return (capital gains and losses and reinvestment income).

Yield to Maturity (YTM)

The yield to maturity (YTM) is the annualized discount rate that equates the present value of a bond's cash flows to its current market price plus accrued interest. The YTM is essentially the internal rate of return (IRR) of the bond's cash flows assuming that the bond will be held till maturity.

$$\text{Bond price} = \frac{CPN_1}{\left(1+\frac{YTM}{2}\right)} + \frac{CPN_2}{\left(1+\frac{YTM}{2}\right)^2} + \frac{CPN_{2N} + \text{Par}}{\left(1+\frac{YTM}{2}\right)^{2N}}$$

where:
Bond price = Full price including accrued interest.
CPN_t = The semiannual coupon payment received after t semiannual periods.
N = Number of years to maturity.
YTM = Yield to maturity.

Example 2: Computing the YTM for Semiannual-Pay Coupon Bonds

Compute the YTM of a 10-year, $1,000 par bond with an 8% coupon rate that makes semiannual coupon payments given that its current price is $925.

Solution

PV = -$925; N = 10*2 = 20; PMT = $40; FV = $1,000; CPT I/Y; I/Y ➔ 4.581%

The calculated yield of 4.581% is the periodic or *semiannual discount rate*. This semiannual discount rate is multiplied by two to determine the bond equivalent yield (BEY), which is also known as the semiannual YTM or semiannual-pay YTM.

YTM = 4.581% * 2 = 9.16%

- If you are given the bond equivalent yield, make sure that you divide the rate by two to obtain the semiannual rate when working with semiannual-pay bonds.
- For bonds that pay coupon annually, the *annual-pay YTM* is simply the IRR of the bond's cash flow stream.

Table 1 summarizes some very important relationships

Table 1

Bond selling at	Relationship
Par	Coupon rate = Current yield = Yield to maturity
Discount	Coupon rate < Current yield < Yield to maturity
Premium	Coupon rate > Current yield > Yield to maturity

Example 3: Computing YTM for Annual-Pay Coupon Bonds

Compute the YTM for a 10-year, $1,000 par bond that pays an 8% annual coupon given that its current price is $925.

Solution

PV = -$925; N = 10; PMT = $80; FV = $1,000; CPT I/Y; I/Y ➔ 9.178%

The bond's YTM equals the annual discount rate of 9.178%

Example 4: Calculating YTM for Zero-Coupon Bonds

A 4-year zero-coupon bond is currently priced at $786. Determine the annual-pay and semiannual-pay YTM on the bond.

Semiannual-pay YTM

$\$786 * (1 + \text{semiannual discount rate})^8 = \$1,000$

PV = -$786; N = 4*2 = 8; FV = $1,000; PMT = 0; CPT I/Y; I/Y ➔ 3.056%

Semiannual-pay YTM on BEY basis = 3.056% * 2 = 6.11%

Annual-pay YTM

$\$786 * (1 + \text{annual discount rate})^4 = \$1,000$

PV = -$786; N = 4, FV = $1,000; PMT = 0; CPT I/Y; I/Y ➔ 6.205%

Annual discount rate = 6.205% = Annual-pay YTM

Limitations of the YTM Measure

The yield to maturity considers coupon income, capital income, and the timing of cash flows. Further, it also considers reinvestment income in that it assumes that interim cash flows will be reinvested at the yield to maturity. Practically speaking however, it can never be guaranteed that the bond's interim cash flows will be successfully invested at a rate equal to the YTM.

It is important to remember that an investor will only realize the yield to maturity calculated at the time of purchase if:

- The interim cash flows are reinvested at the stated yield to maturity. The investor faces the risk that future interest rates could be lower than the YTM at the time of purchase. This is known as *reinvestment risk.*
- The bond is held till maturity. If the investor sells the bonds prior to maturity she faces the risk of receiving less than the purchase price, which would resul in a lower return on investment than the YTM. This is known as *interest rate risk.*

LOS 65c: Explain the importance of reinvestment income in generating the yield computed at the time of purchase, calculate the amount of income required to generate that yield, and discuss the factors that affect reinvestment risk. Vol 5, pg 393-398

Reinvestment income is important because if the reinvestment rate is lower than the YTM calculated at the time of bond purchase, the actual yield realized on the investment will be lower than the YTM.

We already know that the three sources of return from a fixed income investment are coupon payments, capital gains and reinvestment income. In order to determine the amount of reinvestment income required from an investment to attain a certain yield, we subtract the dollar amount of coupon payments and capital gains from the total dollar return required from the investment to obtain the yield.

Because this is a semiannual-pay bond, the number of discounting periods equals 10 (5*2) and the periodic discount rate equals 4% (8%/2).

Example 5: Calculating Reinvestment Income Expected from a Bond Investment

Suppose we purchase a 5-year, 7%, $1,000 par bond that makes semiannual coupon payments for $925. How much reinvestment income must be generated to give us a yield to maturity of 8%?

Solution

First, we must convert the yield to maturity (which is stated on BEY basis) into the semiannual discount rate by dividing it by 2:

Semiannual discount rate = YTM/2 = 4%

In order to attain a YTM of 8% on an investment of $925, we will need total future dollars in 5 years equal to:

$\$925 * (1 + 0.04)^{10} = \$1,369.23$; or

PV = -$925; I/Y = 4; N = 10; CPT FV; FV ➔ $1,369.23

Therefore we require a total dollar return over the term of the bond equal to $444.23 ($1,369.23 - $925).

This total return of $444.23 is composed of:

- A capital gain of $75 ($1,000 that will be received at maturity minus $925 paid for the bond).
- Total coupon payments of $350 (semiannual payments of $35 for 5 years).
- Reinvestment income.

Therefore the reinvestment income required from this bond in order to realize a YTM of 8% equals $19.23 ($444.23 - $75 - $350)

Factors that Affect Reinvestment Risk. Vol 5, pg 387-398

- For a given yield to maturity and coupon rate, the *longer* the term to maturity, the *more* the bond's total dollar return depends on reinvestment income, so *greater* the reinvestment risk.
- For a coupon bond with a given YTM and term to maturity, the *higher* the coupon rate, the *higher* the proceeds from the reinvestment of coupon payments, and *higher* the reliance on reinvestment income.
- Given the term and yield to maturity, bonds selling at a premium will be *more* reliant on reinvestment income than those selling at a discount. For discount bonds, the capital gain (from receiving par at maturity and paying less than par to purchase the bond) will contribute to the total yield.
- Zero-coupon bonds do *not* depend on reinvestment income at all and therefore have zero reinvestment risk.

LOS 65d: Compute and interpret the bond equivalent yield of an annual-pay bond, and the annual-pay yield of a semiannual-pay bond. **Vol 5, pg 399**

U.S. bonds make semiannual coupon payments, while many government bonds in Europe make coupon payments annually. Given the yield to maturity of an annual-pay bond, its bond equivalent yield is computed by computing the semiannual YTM and multiplying it by two.

Example 6: Comparing Semiannual and Annual-Pay Bonds

Which of the following bonds offer a better return?

Bond A: An annual-pay bond with a YTM of 7%.
Bond B: A semiannual-pay bond with a bond equivalent yield of 6.8%.

Solution

We can compare the yields on the bonds on a BEY basis or on an annual-pay yield basis.

Comparison on BEY basis:

To convert Bond A's (the annual-pay bond's) yield to a bond equivalent yield:

First find the semiannual YTM:

$$\text{Semiannual YTM} = [(1+\text{Yield on annual-pay bond})^{0.5} - 1] = (1 + 0.07)^{0.5} - 1 = 3.441\%.$$

Then multiply the semiannual YTM by two:

$$\text{BEY} = 3.441\% * 2 = 6.88\%.$$

Formula to convert annual-pay YTM to BEY:

$$\text{BEY of annual-pay bond} = 2\,[(1+\text{Yield on annual-pay bond})^{0.5} - 1]$$

- The BEY of Bond A (6.88%) is greater than the BEY of Bond B(6.8%). Bond A offers a higher return.

Comparison on annual-pay yield basis:

Bond's B annual-pay yield is calculated by:

First, dividing the BEY by two to determine the semiannual yield:

$$\text{Semiannual yield} = 6.8\%/2 = 3.4\%$$

And then compounding the resulting semiannual YTM for 2 periods:

$$= (1 + \text{semiannual yield})^{2} - 1] = 1.034^{2} - 1 = 6.92\%.$$

Notice that the bond equivalent yield (6.88%) will always be *lower* than the annual-pay bond's yield to maturity (7%).

Formula to convert BEY into annual-pay YTM:

$$\text{Annual-pay yield} = \left[\left(1 + \frac{\text{Yield on bond equivalent basis}}{2}\right)^2 - 1\right]$$

- Notice that the yield on an annual-pay basis (6.92%) is always *greater* than the bond equivalent yield (6.8%).
- Bond A offers a better annual-pay yield (7%) than Bond B (6.92%). Once again, we reach the conclusion that Bond A offers a more attractive return.

Notice that the yield on an annual-pay basis (6.92%) is always *greater* than the bond equivalent yield (6.8%).

Yield to Call. Vol 5, pg 400-401

The yield to call is calculated for callable bonds that are trading at a premium. To calculate the *yield to first call*, the final payment from the cash flow stream (FV) is set to the first call price, and the number of periods (N) is set to the number of discounting periods remaining till the first call date.

The *yield to first par call* is calculated by setting the final payment (FV) as par and the number of periods (N) to the number of discounting periods till the call date at which the call price equals par for the first time.

The yield to call *does* consider all three sources of investment income for fixed income instruments, but it assumes that interim cash flows will be reinvested at the yield to call. Moreover, the yield to call assumes that:

- The investor will hold the bond till the call date.
- The issuer will call the bond at the particular call date.

These two assumptions are unrealistic. Further, comparisons between yields to call and yields to maturity are meaningless because coupon payments will not be made once the bond is called, so the investment horizons that the two measures relate to may be different.

Example 7: Computing YTM, YTC and Yield to First Call

For example, consider a 10-year, 8%, semiannual-pay bond with a price of $121 that cannot be called for 3 years. After 3 years the issuer can call the bond at a price of $113, and after 6 years the bond can be called at par. Calculate the YTM, YTC and yield to first par call.

Solution

The YTM can be calculated as:

PV = -$121; N = 20; PMT = $4; FV = $100; CPT I/Y; I/Y→ 2.636%

This calculated semiannual discount rate must be multiplied by two to determine the YTM on BEY basis. Therefore:

YTM = 2.636% * 2 = 5.27%

To compute the yield to first call (YTFC) we use the number of semiannual discounting periods till the first call date (3*2 = 6) for N and the first call price ($113) for FV:

PV = -$121; N = 6; PMT = $4; FV = $113; CPT I/Y; I/Y➔ 2.265%

The YTFC on BEY basis equals 2.265%* 2 = 4.53%

To compute the yield to first par call (YTFPC) we use the number of semiannual discounting periods remaining till the first par call date (12) for N and the par value ($100) for FV:

PV = -$121; N = 12; PMT = $4; FV = $100; CPT I/Y; I/Y➔ 2.0127%

The YTFPC on a BEY basis equals 2.0127%* 2 = 4.025%

Yield to Put. Vol 5, pg 402

The yield to put is the interest rate that equates the present value of cash flows till the first put date to the price of the bond plus accrued interest. Limitations of the yield to put calculation include the following assumptions:

- Reinvestment of interim cash flows at the yield to put.
- The bond will be put at the first put date.

Yields to put are usually computed for putable bonds that are trading at a discount.

Example 8: Computing YTM and YTP

Consider a 10-year, 8% semiannual-pay bond, that has a par value of $1,000 and currently trades at a full price of $837.50 The first put opportunity occurs in 5 years when the bond can be put at par. Calculate the YTM and YTP for this bond.

Solution

The YTM is calculated as:

PV = -$837.50; N = 20; PMT = $40; FV = $1,000; CPT I/Y; I/Y➔ 5.342%

YTM on BEY basis equals 5.342%* 2 = 10.68%

The YTP is calculated as:

PV = -$837.50, N = 10, PMT = $40, FV = $1,000; CPT I/Y; I/Y➔ 6.232%

YTP on BEY basis equals 6.23% * 2 = 12.46%

Yield to Worst

After calculating the yield to every call and put date, and the yield to maturity, the yield to worst is defined as the lowest of all these calculated yields. It is supposed to indicate the worst yield that the investor could realize on her investment, but it does not recognize that each yield calculation has a different exposure to reinvestment risk because it is based on a different time horizon. For example, the yield to maturity assumes that the bond is held till maturity, while the yield to first call assumes that the bond is held till the first call date. The yield to maturity is calculated over a longer time horizon, which entails a higher level of reinvestment risk. Further, the yield to worst does not identify the potential return over a specified time horizon.

Yield to Refunding

This yield measure is used for bonds that are currently callable, and are trading at a premium to par, (which makes it is feasible for the issuer to call them) but cannot be called due to the existence of certain bond covenants that preclude the financing of a call through a refunding. The number of periods used in calculating this yield measure equals number of discounting periods till the refunding protection ends. Remember that covenants that prohibit refunding only prevent issuers from financing the call through the issuance of lower coupon debt; companies are free to fund the call through other sources.

Cash Flow Yield

Mortgage and asset-backed securities are backed by pools of loans or receivables. There is no way to accurately predict the rate at which the principal will be repaid on these investments. If interest rates rise, the prepayment rate would be lower than expected, and if interest rates were to fall, the prepayment rate would be higher than expected. Usually, an assumption is made about the rate at which prepayments will occur over the term of the security. The yield that equates the present value of the projected cash flows, using a prepayment rate assumption, to the price plus accrued interest is known as the cash flow yield of the bond.

Typically, cash flows for MBS and ABS occur monthly. Bringing this monthly yield measure to a bond equivalent basis involves two steps:

1. Compute the semiannual yield on a six-month basis by compounding the monthly yield for 6 months.

2. Compute the yield on a bond equivalent basis by multiplying the semiannual yield by two.

$$\text{BEY} = [(1 + \text{monthly CFY})^6 - 1] * 2$$

Limitations of the cash flow yield Vol 5, pg 403

- The projected cash flows are assumed to be reinvested at the cash flow yield. This assumption regarding the reinvestment rate has even more significance for MBS and ABS because unlike other fixed income instruments, interest AND interim principal payments are assumed to be reinvested at the calculated cash flow yield.
- The security is assumed to be held till maturity.
- The cash flow yield is dependent on realizing the projected cash flows according to the assumed prepayment rate. If the actual prepayment rate turns out to be different, the calculated cash flow yield will not be realized.

Spread/Margin Measures for Floating-Rate Securities. Vol 5, pg 403-406

The coupon rate for floating-rate securities depends on a reference rate, which usually is different at every reset date. It is not possible to estimate coupon rates going forward because reference rates are not known. Because the yield to maturity cannot be calculated without knowledge of future coupon payments, 'margin' measures are typically used to evaluate floating-rate securities.

Spread for life is a measure of potential return that accounts for the quoted margin as well as the accretion of any discount and the amortization of any premium (as the case may be) over the security's remaining life.

The limitation of this yield measure for floating-rate securities is that it does not consider the level of coupon rates or the time value of money.

Discount Margin estimates the yearly margin over the reference rate that the investor can expect to earn over the life of the security. It determines future cash flows assuming that reference rates will not change and then uses trial and error to find the margin above the reference rate that equates the present value of the cash flows to the security's current price.

- If the bond is selling for par, the discount margin equals the quoted margin.
- If the bond is selling at a premium, the discount margin is *lower* than the quoted margin.
- If the bond is selling for a discount, the discount margin is *higher* than the quoted margin.

The limitations of the discount margin are that it assumes that the reference rate will not change over time, and it does not account for any floor or cap that may be embedded in the security.

Yield on T-Bills Vol 5, pg 406-407

The yield on Treasury bills is determined by two variables:

1. The settlement or transaction price per $1 of face value, p.
2. The number of days to maturity, N. This equals the number of days between the settlement date and the maturity date.

The yield on a discount basis, denoted by *d* is calculated as:

$$d = (1\text{-}p)\,\frac{360}{N}$$

The yield quoted on a discount basis is not a meaningful measure of return from holding a Treasury bill for two reasons:

- The yield is annualized on a 360-day year; not a 365-day year.
- The measure is based on a maturity value investment; not on the actual amount invested.

LOS 65e: Describe the methodology for computing the theoretical Treasury spot rate curve and compute the value of a bond using spot rates. Vol 5. pg 408-413

The Treasury spot rate curve is derived through the 'bootstrapping' process. The process begins by first deriving the Treasury yield curve, which illustrates the relationship between YTMs on Treasuries and their terms to maturity.

On-the-run Treasuries are not available for every investment horizon. Therefore, the YTM for on-the-run Treasury issues is available only for certain maturities. For example, if there are on-the-run Treasuries with 2, 5, 7, and 10 years to maturity available, their YTMs can be computed from market prices. The yields for horizons for which on-the-run Treasury securities are not available are estimated through a variety of interpolation methods, the simplest of which is linear interpolation. Linear interpolation evenly distributes the difference in yields over the time period between two maturities. For example, if the yield on the 7-year Treasury is 4% and that on the 10-year Treasury is 5%, linear interpolation would compute the 8-year yield to be 4.33%, and the 9-year yield to be 4.67%.

Once the yield on a variety of maturities has been calculated, simple algebra is used to 'bootstrap' the spot rate curve.

A yield curve that assumes that all coupon-bearing securities are trading at par is known as the par yield curve.

The following table provides YTMs on three Treasury bonds with different maturities. We have assumed that the prices of the coupon-bearing bonds equal their par value. This is usually not the case, but we use par values to make the subsequent calculations easier to follow.

Maturity	YTM on BEY Basis	Coupon	Price
6 months	4%	Nil	$980.39
1 year	5%	$25	$1,000
1.5 years	6%	$30	$1,000

The 6-month spot rate equals the yield to maturity on the zero-coupon bond. The yield on the zero-coupon bond is given as 4% on BEY basis.

See Example 1 in Reading 64 for an illustration on the use of individual spot rates to value a bond.

The 1-year bond gives out cash flows at two points in time- after 6 months, and after 1 year. We know that the price of a bond equals the sum of the present values of its cash flows, with each cash flow discounted at the individual spot rate that coincides with its timing. The six-month spot rate is used to discount the cash flow that occurs after 6 months. The 1-year spot rate is unknown, but can be easily calculated by equating the present value of the bond's cash flows to the price of the bond ($1,000)

$$\frac{25}{1 + 0.04/2} + \frac{1{,}025}{(1 + {}^{x}/2)^2} = 1{,}000$$

$$(1+\frac{x}{2})^2 = \frac{1{,}025}{975.49}$$

$$\frac{x}{2} = 0.02506$$

$$x = 0.05013$$

The 1-year spot rate equals 5.013% on a BEY basis.

Now that we have know the 6-month and the 1-year spot rate, the 18-month spot rate can be calculated by equating the sum of the present values of the 18-month bond's cash flows to its price ($1,000).

$$\frac{30}{\left(1+\frac{0.04}{2}\right)} + \frac{30}{\left(1+\frac{0.05013}{2}\right)^2} + \frac{1{,}030}{\left(1+\frac{x}{2}\right)^3} = 1{,}000$$

$$\frac{1{,}030}{\left(1+\frac{x}{2}\right)^3} = 942.037$$

$$\frac{x}{2} = \left(\frac{1030}{942.037}\right)^{1/3} - 1 \Rightarrow \ x = 0.06041$$

The 18-month spot rate equals 6.04% on a BEY basis.

The spot rates obtained through the bootstrapping process illustrate the term structure of default-free spot rates at the point in time that the bond price quotes are taken from. You should also remember that:

- When the par yield curve is *upward* sloping, the theoretical spot rate curve will lie *above* the par yield curve.
- When the par yield curve is *downward* sloping, the theoretical spot rate curve will lie *below* the par yield curve.
- When the yield curve is *flat*, the spot rate curve is the *same* as the yield curve.

LOS 65f: Differentiate between the nominal spread, the zero-volatility spread, and the option-adjusted spread. **Vol 5, pg 413-418**

Suppose we observe the following yields on a Treasury and a non-Treasury bond with 10-year terms to maturity:

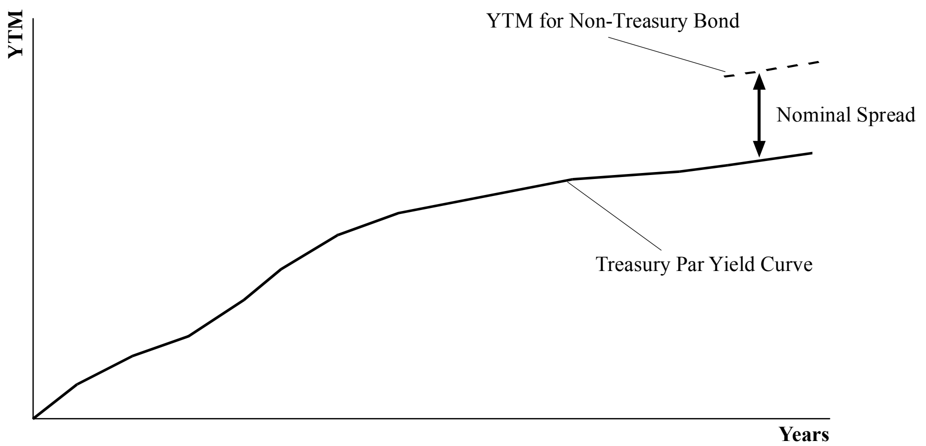
Issue	Coupon	Price	YTM
Treasury	11%	$100.00	11.00%
Non-Treasury	14%	$109.25	12.34%

The difference in the YTMs between the two, i.e. 134 basis points, is known as the **nominal spread**. The nominal spread accounts for the higher credit risk, liquidity risk and option risk (if the bond has embedded options) in the non-Treasury bond compared to the Treasury bond.

Limitations of the nominal spread as a yield spread measure:

1. The yield to maturity ignores the shape of the spot rate curve. It is appropriate to use spread measures based on the YTM only if the yield curve is flat (essentially because the yield curve would then be identical to the spot rate curve).
2. The nominal spread does not consider any embedded options that may alter the cash flows from a non-Treasury bond, and materially affect the value of the security.

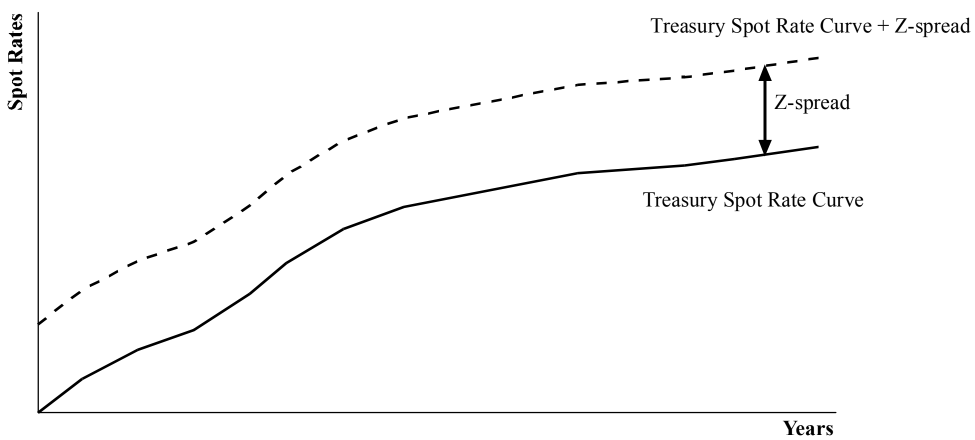
Figure 1: Nominal Spread



The **zero-volatility spread** or **Z-spread** is the static or constant spread that makes the present value of the cash flows from a non-Treasury bond, when discounted at Treasury spot rates plus the spread, equal to the non-Treasury bond's price. In other words, the Z-spread is the spread an investor would earn above the entire Treasury spot rate curve by investing in a non-Treasury bond as opposed to a Treasury bond, and holding the instrument till maturity.

The difference between the nominal spread and the Z-spread is that the nominal spread is a spread on any *one* point on the *Treasury yield* curve, while the Z-spread is a *constant* spread over the *entire Treasury spot rate curve.*

Figure 2: Z-Spread



The Z-spread measures the additional compensation for the liquidity risk, credit risk and any option risk in a non-Treasury security when the benchmark is the Treasury spot rate curve. If the non-Treasury issuer's own spot rate curve is used as the benchmark, the Z-spread measures only the liquidity and option risk. Therefore, when a Z-spread is given, the benchmark spot rate curve must always be stated.

Two factors determine the significance of divergence between the Z-spread and the nominal spread:

- When the Treasury spot rate curve is flat, the Z-spread and the nominal spread are equal because the same benchmark rates are used to discount the bond's cash flows. When the Treasury spot rate curve is flat, it is identical to the Treasury yield curve.
- As the Treasury spot rate curve steepens, the difference between the nominal spread and Z-spread widens.
- The difference between the Z-spread and the nominal spread is greater for securities that repay principal over time as opposed to in a single bullet payment at maturity. For example, the difference between the two spread measures is greater for mortgage and asset-backed securities than for straight bonds.

The Z-spread successfully counters the first limitation of the nominal spread because it is based on the Treasury spot rate curve; not the Treasury yield curve. However, it does not account for any options embedded in the bond's structure. Consequently, an investor who relies on the Z-spread or the nominal spread to evaluate fixed income investments may not be adequately compensated for the option risk in securities with embedded options.

LOS 65g: Describe how the option-adjusted spread accounts for the option cost in a bond with an embedded option. **Vol 5, pg 418-421**

The **option-adjusted spread (OAS)** counters both the limitations of the nominal spread- it is a spread over a spot rate curve, and it accounts for embedded options. To determine the OAS, first the dollar value of the embedded option is calculated and then the bond's dollar price is adjusted for the cost of the option. The resulting 'option-free' price is then converted into a yield measure that is 'option-adjusted'.

The Z-spread ignores the fact that a change in interest rates can alter the cash flows from bonds with embedded options. It assumes that there is no volatility of interest rates. Such an environment renders embedded options worthless.

The OAS adjusts the Z-spread for the option risk in the bond. The OAS of a bond with embedded options is comparable to the Z-spread of an option-free bond.

Z-spread = OAS + Option cost; and OAS = Z-spread - Option cost

- If option cost is *positive* in terms of yield, it means that the issuer is making payments at a spread greater than the spread on an option-free bond (the Z-spread). This implies that the issuer has purchased an option to alter the bond's cash flows, as is the case with a callable bond.

- If option cost is *negative* in terms of yield, it means that the issuer is making payments at a spread that is lower than the spread on an option-free bond. This occurs when the issuer's interest payments have been subsidized by selling an option to bondholders, as is the case with putable bonds.

Risks Captured by the Various Spread Measures

Spread Measure	Benchmark	Reflects compensation for
Nominal	Treasury yield curve	Credit risk, option risk, liquidity risk
Zero-volatility	Treasury spot rate curve	Credit risk, option risk, liquidity risk
Option-adjusted	Treasury spot rate curve	Credit risk, liquidity risk

LOS 65h: Explain a forward rate, and compute spot rates from forward rates, forward rates from spot rates, and the value of a bond using forward rates.

Vol 5, pg 421-431

Earlier, we illustrated how spot rates can be extrapolated from yield curves using algebra (bootstrapping). Now we will use the same concept to derive forward rates from the spot rate curve. Forward rates can be described as the market's current estimate of future spot rates. We will use also use the arbitrage principal in our derivation- two portfolios with identical cash flows and identical risks should have the same value today, all other factors constant.

Consider an investor who has a 1-year investment horizon and is faced with the following alternatives:

- Purchase a one-year T-bill today. The 1-year spot rate today (yield on the zero-coupon 1-year T-bill) is given as 4.6%.
- Purchase a 6-month T-bill now and upon its expiration, purchase another 6-month T-bill. The 6-month spot rate today (yield on the first 6-month T-bill) is given as 4%.

Notice that in this example we work with 6-month periods. The yields on the bonds have been expressed on a bond-equivalent basis- the semiannual rate has been multiplied by two.

The investor will be indifferent between these two options if she knows that the return from taking either option with be the same. However, she does not know the return that will be offered after 6 months on the second 6-month T-bill. We can calculate the return required (to make her indifferent between the two options) on a 6-month investment in a T-bill, 6 months from now by using the spot rates available today for the 6-month and the 1-year T-bill:

0.04/2 is the effective six-month discount rate on the 6-month T-Bill. 0.04 is the yield on bond equivalent basis.

If she invests $100 today in the 6-month T-bill her return would be:

$$100 * \left(1 + \frac{0.04}{2}\right)^1 = \$102$$

0.046/2 is the effective six-month discount rate on the 1 year T-Bill. 0.046 is the yield on bond equivalent basis.

If she invests in 1-year T-bill, she will end up with:

$$100 * \left(1 + \frac{0.046}{2}\right)^2 = \$104.65$$

The investor would be indifferent between the two strategies if they offer her an identical return. In order to end up with \$104.65 using the rollover strategy, her return on the second 6 month T-bill must equal 5.2% on BEY basis. This figure is calculated as:

$$102 * \left(1 + \frac{x}{2}\right) = \$104.6529$$

$$\frac{104.6529}{102} - 1 = \text{ 5.2\% on BEY basis}$$

If the yields presented are on BEY basis:

$$\left(1 + \frac{\text{6-mth spot rate}}{2}\right)\left(1 + \frac{\text{6-mth forward rate 6 mths from now}}{2}\right) = \left(1 + \frac{\text{12-mth spot rate}}{2}\right)^2$$

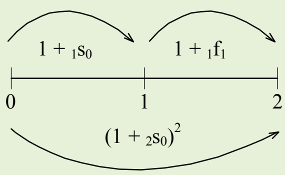
Example 9: Computing Forward Rates

The current 1-year spot rate is 5%, 2-year spot rate is 5.25% and 3-year spot rate is 5.55%. Calculate the 1-year forward rate 1 year from now and 2 years from now.

Solution

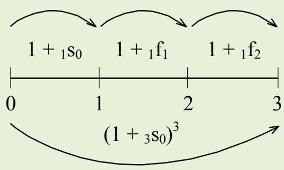
Calculation of 1-year forward rate 1 year from today:

$(1 + {}_1S_0)(1 + {}_1f_1) = (1 + {}_2S_0)^2$
$(1 + 0.05)(1 + {}_1f_1) = (1 + 0.0525)^2$



$${}_1f_1 = \frac{1.0525^2}{1.05} - 1 = 5.5\%$$

Calculation of 1-year forward rate 2 years from today:

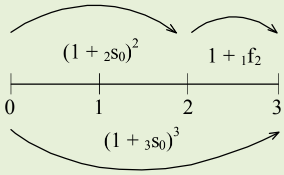


$(1 + {}_1S_0)(1 + {}_1f_1)(1 + {}_1f_2) = (1 + {}_3S_0)^3$

$(1 + {}_1S_0)(1 + {}_1f_1)$ simply equals the compounding factor for an investment for 2 years at the two-year spot rate. The equation above can therefore be modified to:

$(1 + {}_2S_0)^2\,(1 + {}_1f_2) = (1 + {}_3S_0)^3$

$${}_1f_2 = \frac{1.0555^3}{1.0525^2} - 1 = 6.15\%$$



Forward rates are market estimates of future spot rates.

${}_1S_0$ = 1 period spot rate today (t = 0).

${}_xS_0$ = x-period spot rate today (t = 0).

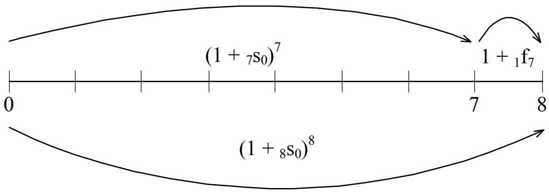
${}_2f_5$ = 2-period forward rate 5 periods from today.

Let's summarize what we have learned so far about the relationship between multi-period spot rates and forward rates:

- $(1 + {}_1s_0)(1 + {}_1f_1) = (1 + {}_2s_0)^2$
- $(1 + {}_2s_0)^2\,(1 + {}_1f_2) = (1 + {}_3s_0)^3$

Therefore we can calculate the one-period forward rate 7 years from now using the 8-year and the 7-year spot rates:

$$(1 + {}_7s_0)^7\,(1 + {}_1f_7) = (1 + {}_8s_0)^8$$



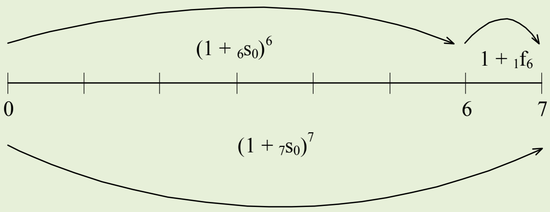
Example 10: Calculating Forward Rates

Calculate the 1-year forward rate 6 years from today if the 6-year spot rate is 6.25% and the 7-year spot rate is 6%.

Solution:

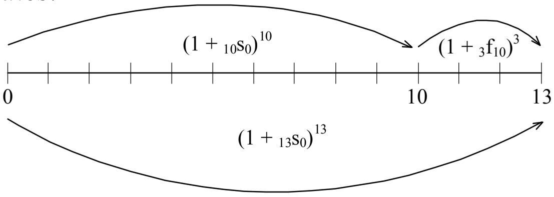
$$(1 + {}_6s_0)^6\,(1 + {}_1f_6) = (1 + {}_7s_0)^7$$

$$(1 + {}_1f_6) = \frac{(1.06)^7}{(1.0625)^6} \Rightarrow {}_1f_6 = 4.51\%$$



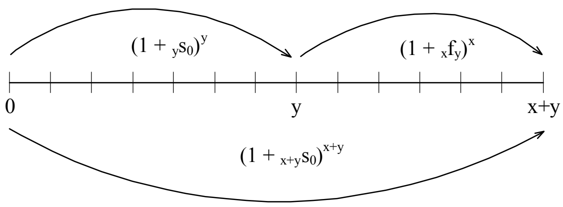
We can also calculate multi-period forward rates using multi-period spot rates. For example, we can use the following equation to calculate the 3-period forward rate 10 periods from now using the 10-year and the 13-year spot rates:

$$(1 + {}_{10}s_0)^{10}\,(1 + {}_3f_{10})^3 = (1 + {}_{13}s_0)^{13}$$



To calculate the x-period forward rate y periods from today, simply remember the following formula:

$$(1 + {}_ys_0)^y\,(1 + {}_xf_y)^x = (1 + {}_{x+y}s_0)^{x+y}$$



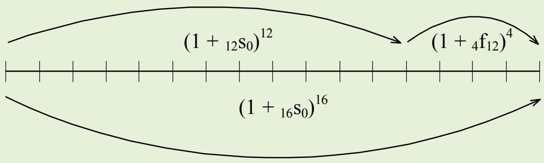
Example 11: Calculating Multi-Period Forward Rates

Calculate the 4-year forward rate 12 years from today if the 12-year spot rate is 4.5% and the 16-year spot rate is 4.6%.

Solution

$$(1+{}_{12}s_0)^{12}(1+{}_4f_{12})^4=(1+{}_{16}s_0)^{16}$$

$$(1+{}_4f_{12})^4=\frac{(1.046)^{16}}{(1.045)^{12}} \Rightarrow {}_4f_{12}=4.9\%$$



Example 12: Valuing Bonds Using Forward Rates

The current 1-year forward rate is 4%, the 1-year forward rate 1 year from now is 4.25% and the 1-year forward rate 2 years from today is 4.3%. Calculate the value of a $1,000 par, annual-pay coupon bond that has a coupon rate of 4%.

Solution

$$\frac{40}{1+{}_1f_0}+\frac{40}{(1+{}_1f_0)(1+{}_1f_1)}+\frac{1{,}040}{(1+{}_1f_0)(1+{}_1f_1)(1+{}_1f_2)}$$

$$\frac{40}{1+0.04}+\frac{40}{(1+0.04)(1+0.0425)}+\frac{1{,}040}{(1+0.04)(1+0.0425)(1+0.043)}=\$995.04$$

Even though we include the 1-year forward rate today in this example, this rate really is not a forward rate; it is simply the 1-year spot rate today.