Book 5 Formulae

Bond Coupon

Coupon = Coupon rate × Par value

Coupon Rate (Floating)

Coupon Rate = Reference rate + Quoted margin

Coupon Rate (Inverse Floaters)

Coupon rate = K – L× (Reference rate)

Callable Bond Price

Price of a callable bond = Value of option-free bond – Value of embedded call option

Putable Bond Price

Price of a putable bond = Value of option-free bond + Value of embedded put option

Dollar Duration

Dollar duration = Duration × Bond value

Nominal spread

Nominal spread (Bond Y as the reference bond) = Yield on Bond X – Yield on Bond Y

Relative Yield spread

$$\text{Relative yield spread} = \frac{\text{Yield on Bond X} - \text{Yield on Bond Y}}{\text{Yield on Bond Y}}$$

Yield Ratio

$$\text{Yield ratio} = \frac{\text{Yield on Bond X}}{\text{Yield on Bond Y}}$$

After-Tax Yield

After-tax yield = Pretax yield × (1- marginal tax rate)

Taxable-Equalent Yield

$$\text{Taxable-equivalent yield} = \frac{\text{Tax-exempt yield}}{(1 - \text{marginal tax rate})}$$

Bond Value

$$\text{Bond Value} = \frac{\text{Maturity value}}{(1+i)^{\text{years till maturity} \times 2}}$$

where i equals the semiannual discount rate

Current Yield

$$\text{Current yield} = \frac{\text{Annual cash coupon}}{\text{Bond price}}$$

Bond Price

$$\text{Bond price} = \frac{CPN_1}{\left(1+\frac{YTM}{2}\right)} + \frac{CPN_2}{\left(1+\frac{YTM}{2}\right)^2} + \frac{CPN_{2N} + Par}{\left(1+\frac{YTM}{2}\right)^{2N}}$$

where:
Bond price = Full price including accrued interest.
CPN_t = The semiannual coupon payment received after t semiannual periods.
N = Number of years to maturity.
YTM = Yield to maturity.

Formula to Convert BEY into Annual-Pay YTM:

$$\text{Annual-pay yield} = \left[\left(1 + \frac{\text{Yield on bond equivalent basis}}{2}\right)^2 - 1\right]$$

Formula to Convert Monthly Cash Flow Yield into BEY

$$BEY = [(1 + \text{monthly CFY})^6 - 1] \times 2$$

Discount Basis Yeild

$$d = (1-p)\frac{360}{N}$$

Z-Spread

Z-spread = OAS + Option cost; and OAS = Z-spread - Option cost

Duration

$$\text{Duration} = \frac{V_- - V_+}{2(V_0)(\Delta y)}$$

where:
Δy = change in yield in decimal
V_0 = initial price
V_- = price if yields decline by Δy
V_+ = price if yields increase by Δy

Portfolio Duration

$$\text{Portfolio duration} = w_1D_1 + w_2D_2 + \ldots + w_ND_N$$

where:
N = Number of bonds in portfolio.
D_i = Duration of Bond i.
w_i = Market value of Bond i divided by the market value of portfolio.

Percentage Change in Bond Price

Percentage change in bond price = duration effect + convexity adjustment
$= \{[-\text{duration} \times (\Delta y)] + [\text{convexity} \times (\Delta y)^2]\} \times 100$

where:
Δy = Change in yields in decimals.

Convexity

$$C = \frac{V_+ + V_- - 2V_0}{2V_0(\Delta y)^2}$$

Price Value of a Basis Point

Price value of a basis point = Duration × 0.0001 × bond value

FRA Payoff

$$\frac{\text{Floating rate at expiration} - \text{FRA rate} \times (\text{days in floating rate}/\ 360)}{1 + [\text{Floating rate at expiration} \times (\text{days in floating rate}/\ 360)}$$

Numerator: Interest savings on the hypothetical loan. This number is positive when the floating rate is greater than the forward rate. When this is the case, the long benefits and expects to receive a payment from the short. The numerator is negative when the floating rate is lower than the forward rate. When this is the case, the short benefits and expects to receive a payment from the long.

Denominator: The discount factor for calculating the present value of the interest savings.

Call Option Payoffs

Option Position	Description	Payoffs	
		$S_T > X$	**$S_T < X$**
		Option holder exercises the option.	Option holder does not exercise the option.
Call option holder	Choice to buy the underlying asset for X	$S_T - X$	0
Call option writer	Obligation to sell the underlying asset for X if the option holder chooses to exercise the option	$-(S_T - X)$	0

Intrinsic Value of a Call Option

Intrinsic value of call = Max $[0, (S_t - X)]$

Put Option Payoffs

Option Position	Description	Payoffs	
		$S_T < X$	**$S_T > X$**
		Option holder exercises the option	Option holder does not exercise the option
Put option holder	Choice to sell the underlying asset for X	$X - S_T$	0
Put option writer	Obligation to buy the underlying asset for X if the option holder chooses to exercise the option	$-(X - S_T)$	0

Moneyness and Intrinsic Value of a Put Option

Moneyness	Current Market Price (S_t) versus Exercise Price (X)	Intrinsic Value Max $[0, (X - S_t)]$
In-the-money	S_t is less than X	$X - S_t$
At-the-money	S_t equals X	0
Out-of-the-money	S_t is greater than X	0

Option Premium

Option premium = Intrinsic value + Time value

Put-Call Parity

$$C_0 + \frac{X}{(1 + R_F)^T} = P_0 + S_0$$

Synthetic Derivative Securities

Strategy	Consisting of	Value	Equals	Strategy	Consisting of	Value
fiduciary call	long call + long bond	$C_0 + \frac{X}{(1 + R_F)^T}$	=	Protective put	long put + long underlying asset	$P_0 + S_0$
long call	long call	C_0	=	Synthetic call	long put + long underlying asset + short bond	$P_0 + S_0 - X/(1+R_F)^T$
long put	long put	P_0	=	Synthetic put	long call + short underlying asset + long bond	$C_0 - S_0 + X/(1+R_F)^T$
long underlying asset	long underlying asset	S_0	=	Synthetic underlying asset	long call + long bond + short put	$C_0 + X/(1+R_F)^T - P_0$
long bond	long bond	$\frac{X}{(1 + R_F)^T}$	=	Synthetic bond	long put + long underlying asset + short call	$P_0 + S_0 - C_0$

Option Value Limits

Option	Minimum Value	Maximum Value
European call	$EC_t \geq 0$	$EC_t \leq S_t$
American call	$AC_t \geq 0$	$AC_t \leq S_t$
European put	$EP_t \geq 0$	$EP_t \leq X/(1 + RFR)^T$
American put	$AP_t \geq 0$	$AP_t \leq X$

Option Value Bounds

Option	Minimum Value	Maximum Value
European Call	$\text{Max}\left[0, S_t - \frac{X}{(1+RFR)^T}\right]$	S_t
American Call	$\text{Max}\left[0, S_t - \frac{X}{(1+RFR)^T}\right]$	S_t
European Put	$\text{Max}\left[0, \frac{X}{(1+RFR)^T} - S_t\right]$	$\frac{X}{(1+RFR)^T}$
American Put	$\text{Max}\left[0, X - S_t\right]$	X

Interest Rate Call Holder's Payoff

$$= \text{Max}(0, \text{Underlying rate at expiration} - \text{Exercise rate}) \left(\frac{\text{Days in underlying Rate}}{360}\right) \times NP$$

where: NP = Notional principal

Interest Rate Put Holder's Payoff

$$= \text{Max}(0, \text{Exercise rate} - \text{Underlying rate at expiration}) \left(\frac{\text{Days in underlying rate}}{360}\right) \times NP$$

where:
NP = Notional principal

Net Payment for a Fixed-Rate-Payer

Net fixed-rate payment$_t$ = (Swap fixed rate - LIBOR$_{t-1}$)×(No. of days/360)×(NP)
where:
NP equals the notional principal.

Real Estate Valuation: Appraisal Price

$$\text{Appraisal price} = \frac{\text{NOI}}{\text{Market cap rate}}$$

Real Estate Valuation: Market Capitalization Rate

$$\text{Market cap rate} = \frac{\text{Benchmark NOI}}{\text{Benchmark transaction price}}$$