Book 1 Formulae

The Future Value of a Single Cash Flow

$FV_N = PV\,(1+r)^N$

The Present Value of a Single Cash Flow

$$PV = \frac{FV}{(1+r)^N}$$

$PV_{\text{Annuity Due}} = PV_{\text{Ordinary Annuity}} \times (1 + r)$
$FV_{\text{Annuity Due}} = FV_{\text{Ordinary Annuity}} \times (1 + r)$

$$PV(\text{perpetuity}) = \frac{PMT}{I/Y}$$

$FV_N = PVe^{r_s * N}$

EAR = $(1 + \text{Periodic interest rate})^N - 1$

Net Present Value

$$NPV = \sum_{t=0}^{N} \frac{CF_t}{(1+r)^t}$$

where
CF_t = the expected net cash flow at time t
N = the investment's projected life
r = the discount rate or appropriate cost of capital

Bank Discount Yield

$$r_{BD} = \frac{D}{F} \times \frac{360}{t}$$

where:
r_{BD} = the annualized yield on a bank discount basis.
D = the dollar discount (face value – purchase price)
F = the face value of the bill
t = number of days remaining until maturity

Holding Period Yield

$$HPY = \frac{P_1 - P_0 + D_1}{P_0} = \frac{P_1 + D_1}{P_0} - 1$$

where:
P_0 = initial price of the investment.
P_1 = price received from the instrument at maturity/sale.
D_1 = interest or dividend received from the investment.

Effective Annual Yield

$EAY = (1 + HPY)^{365/t} - 1$

where:
HPY = holding period yield
t = numbers of days remaining till maturity

$HPY = (1 + EAY)^{t/365} - 1$

Money Market Yield

$$R_{MM} = \frac{360 \times r_{BD}}{360 - (t \times r_{BD})}$$

$R_{MM} = HPY \times (360/t)$

Bond Equalent Yield

BEY = [(1 + EAY) ^ 0.5 - 1]

Population Mean

$$\mu = \frac{\sum_{i=1}^{N} x_i}{N}$$

Where,
x_i = is the *i*th observation.

Sample Mean

$$\bar{X} = \frac{\sum_{i=1}^{n} x_i}{n}$$

Geometric Mean

$$1 + R_G = \sqrt[T]{(1+R_1)\times(1+R_2)\times \ldots \times(1+R_T)} \quad \text{OR} \quad G = \sqrt[n]{X_1 X_2 X_3 \ldots X_n}$$

with $X_i \geq 0$ for $i = 1, 2, \ldots, n$.

$$R_G = \left[\prod_{t=1}^{T}(1+R_t)\right]^{\frac{1}{T}} - 1$$

Harmonic Mean

Harmonic mean: $\bar{X}_H = \dfrac{N}{\sum_{i=1}^{N} \frac{1}{x_i}}$ with $X_i > 0$ for $i = 1, 2, \ldots, N$.

Percentiles

$$L_y = \frac{(n+1)\,y}{100}$$

where:
y = percentage point at which we are dividing the distribution
L_y = location (L) of the percentile (P_y) in the data set sorted in ascending order

Range

Range = Maximum value - Minimum value

Mean Absolute Deviation

$$\text{MAD} = \frac{\sum_{i=1}^{n} \left| X_i - \bar{X} \right|}{n}$$

Where:
n = number of items in the data set
$\bar{X}$ = the arithmetic mean of the sample

Population Variance

$$\sigma^2 = \frac{\sum_{i=1}^{N} (X_i - \mu)^2}{N}$$

where:
X_i = observation i
μ = population mean
N = size of the population

Population Standard Deviation

$$\sigma = \sqrt{\frac{\sum_{i=1}^{N} (X_i - \mu)^2}{N}}$$

Sample Variance

$$\text{Sample variance} = \quad s^2 = \frac{\sum_{i=1}^{n} (X_i - \bar{X})^2}{n-1}$$

where:
n = sample size.

Sample Standard Deviation

$$s = \sqrt{\frac{\sum_{i=1}^{n}\left(X_i - \bar{X}\right)^2}{n-1}}$$

Coefficient of Variation

$$\text{Coefficient of variation} = \frac{s}{\bar{X}}$$

where:
s = sample standard deviation
$\bar{X}$ = the sample mean.

Sharpe Ratio

$$\text{Sharpe ratio} = \frac{\bar{r}_p - r_f}{s_p}$$

where:
$\bar{r}_p$ = mean portfolio return
r_f = risk-free return
s_p = standard deviation of portfolio returns

Sample Skewness

$$\text{Sample skewness } (S_K) = \frac{1}{n}\frac{\sum_{i=1}^{n}\left(X_i - \bar{X}\right)^3}{s^3}$$

where:
s = sample standard deviation

Sample Kurtosis

$$\text{Sample kurtosis} = \frac{1}{n}\frac{\sum_{i=1}^{n}\left(X_i - \bar{X}\right)^4}{s^4}$$

where:
s = sample standard deviation

Odds for an event

$$P(E) = \frac{a}{(a+b)}$$

Where the odds for are given as ‘a to b’, then:

Odds for an event

$$P(E) = \frac{b}{(a + b)}$$

Where the odds *against* are given as ‘a to b’, then:

Conditional Probabilities

$$P(A|B) = \frac{P(AB)}{P(B)} \text{ given that } P(B) \neq 0$$

Multiplication Rule for Probabilities

$$P(AB) = P(A | B) \times P(B)$$

Addition Rule for Probabilities

$$P(A \text{ or } B) = P(A) + P(B) - P(AB)$$

For Independant Events

$P(A|B) = P(A)$, or equivalently, $P(B|A) = P(B)$

$P(A \text{ or } B) = P(A) + P(B) - P(AB)$
$P(A \text{ and } B) = P(A) \times P(B)$

The Total Probability Rule

$P(A) = P(AS) + P(AS^c)$

$P(A) = P(A|S) \times P(S) + P(A|S^c) \times P(S^c)$

The Total Probability Rule for *n* Possible Scenarios

$P(A) = P(A|S_1) \times P(S_1) + P(A|S_2) \times P(S_2) + ...+ P(A|S_n) \times P(S_n)$

where the set of events $\{S_1, S_2,..., S_n\}$ is mutually exclusive and exhaustive.

Expected Value

$$E(X) = P(X_1)X_1 + P(X_2)X_2 + \ldots\ldots P(X_n)X_n$$

$$E(X) = \sum_{i=1}^{n} P(X_i)X_i$$

Where:
X_i = one of *n* possible outcomes.

Variance and Standard Deviation

$\sigma^2(X) = E\{[X - E(X)]^2\}$

$$\sigma^2(X) = \sum_{i=1}^{n} P(X_i)\,[X_i - E(X)]^2$$

The Total Probability Rule for Expected Value

1. $E(X) = E(X|S)P(S) + E(X|S^c)P(S^c)$
2. $E(X) = E(X|S_1) \times P(S_1) + E(X|S_2) \times P(S_2) + ... + E(X|S_n) \times P(S_n)$

Where:
$E(X)$ = the unconditional expected value of X
$E(X|S_1)$ = the expected value of X given Scenario 1
$P(S_1)$ = the probability of Scenario 1 occurring
The set of events $\{S_1, S_2, ..., S_n\}$ is mutually exclusive and exhaustive.

Covariance

$\text{Cov}(XY) = E\{[X - E(X)][Y - E(Y)]\}$

$\text{Cov}(R_A,R_B) = E\{[R_A - E(R_A)][R_B - E(R_B)]\}$

Correlation Coefficient

$$\text{Corr}(R_A,R_B) = \rho(R_A,R_B) = \frac{\text{Cov}(R_A,R_B)}{(\sigma_A)(\sigma_B)}$$

Expected Return on a Portfolio

$$E(R_p) = \sum_{i=1}^{N} w_i E(R_i) = w_1 E(R_1) + w_2 E(R_2) + ... + w_N E(R_N)$$

Where:

$$\text{Weight of asset } i = \frac{\text{Market value of investment } i}{\text{Market value of portfolio}}$$

Portfolio Variance

$$\text{Var}(R_p) = \sum_{i=1}^{N}\sum_{j=1}^{N} w_i w_j \text{Cov}(R_i, R_j)$$

Variance of a 2 Asset Portfolio

$$\text{Var}(R_p) = w_A^2\sigma^2(R_A) + w_B^2\sigma^2(R_B) + 2w_A w_B \text{Cov}(R_A, R_B)$$

$$\text{Var}(R_p) = w_A^2\sigma^2(R_A) + w_B^2\sigma^2(R_B) + 2w_A w_B \rho(R_A, R_B)\sigma(R_A)\sigma(R_B)$$

Variance of a 3 Asset Portfolio

$$\mathrm{Var}(R_p) = w_A^2\sigma^2(R_A) + w_B^2\sigma^2(R_B) + w_C^2\sigma^2(R_C)$$
$$+ 2w_A w_B \mathrm{Cov}(R_A, R_B) + 2w_B w_C \mathrm{Cov}(R_B, R_C) + 2w_C w_A \mathrm{Cov}(R_C, R_A)$$

Bayes' Formula

$$P\left(\text{Event} \mid \text{Information}\right) = \frac{P\left(\text{Information} \mid \text{Event}\right) \times P\left(\text{Event}\right)}{P\left(\text{Information}\right)}$$

Counting Rules

The number of different ways that the k tasks can be done equals $n_1 \times n_2 \times n_3 \times \ldots n_k$.

Combinations

$${}_nC_r = \binom{n}{r} = \frac{n!}{(n-r)!(r!)}$$

Remember: The combination formula is used when the order in which the items are assigned the labels is NOT important.

Permutations

$${}_nP_r = \frac{n!}{(n-r)!}$$

Discrete uniform distribution

$F(x) = n \times p(x)$ for the *nth* observation.

Binomial Distribution

$$P\left(X=x\right) = {}_nC_x \left(p\right)^x \left(1-p\right)^{n-x}$$

where:
p = probability of success
1 - p = probability of failure
${}_nC_x$ = number of possible combinations of having x successes in n trials. Stated differently, it is the number of ways to choose x from n when the order does not matter.

The Continuous Uniform Distribution

$P(X < a),\ P(X > b) = 0$

$$P\ (x_1 \leq X \leq x_2) = \frac{x_2 - x_1}{b - a}$$

Confidence Intervals

For a random variable X that follows the normal distribution:
The 90% confidence interval is $\bar{x}$ - 1.65*s* to $\bar{x}$ + 1.65*s*
The 95% confidence interval is $\bar{x}$ - 1.96*s* to $\bar{x}$ + 1.96*s*
The 99% confidence interval is $\bar{x}$ - 2.58*s* to $\bar{x}$ + 2.58*s*

The following probability statements can be made about normal distributions

- Approximately 50% of all observations lie in the interval $\mu \pm (2/3)\sigma$
- Approximately 68% of all observations lie in the interval $\mu \pm 1\sigma$
- Approximately 95% of all observations lie in the interval $\mu \pm 2\sigma$
- Approximately 99% of all observations lie in the interval $\mu \pm 3\sigma$

z-Score

z = (observed value - population mean)/standard deviation = $(x - \mu)/\sigma$

Roy's safety-first criterion

Minimize $P(R_P < R_T)$
where:
R_P = portfolio return
R_T = target return

Shortfall Ratio

$$\text{Shortfall ratio (SF Ratio) or z-score} = \frac{E(R_P) - R_T}{\sigma_P}$$

Continuously Compounded Returns

$EAR = e^{r_{cc}} - 1$ r_{cc} = continuously compounded annual rate

$$HPR_t = e^{r_{cc} \times t} - 1$$

Sampling Error

Sampling error of the mean = Sample mean - Population mean = $\bar{x} - \mu$

Standard Error of Sample Mean when Population variance is Known

$$\sigma_{\bar{x}} = \sigma / \sqrt{n}$$

where:
$\sigma_{\bar{x}}$ = the standard error of the sample mean
σ = the population standard deviation
n = the sample size

Standard Error of Sample Mean when Population variance is Not Known

$$s_{\bar{x}} = \frac{s}{\sqrt{n}}$$

where:
$s_{\bar{x}}$ = standard error of sample mean
s = sample standard deviation.

Confidence Intervals

Point estimate ± (reliability factor × standard error)

where:
Point estimate = value of the sample statistic that is used to estimate the population parameter
Reliability factor = a number based on the assumed distribution of the point estimate and the level of confidence for the interval (1- α).
Standard error = the standard error of the sample statistic (point estimate)

$$\bar{x} \pm z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$$

where:

$\bar{x}$ = The sample mean (point estimate of population mean)
$z_{\alpha/2}$ = The standard normal random variable for which the probability of an observation lying in either tail is $\alpha / 2$ (reliability factor).
$\frac{\sigma}{\sqrt{n}}$ = The standard error of the sample mean.

$$\bar{x} \pm t_{\frac{\alpha}{2}} \frac{s}{\sqrt{n}}$$

where:
$\bar{x}$ = sample mean (the point estimate of the population mean)
$t_{\frac{\alpha}{2}}$ = the t-reliability factor
$\frac{s}{\sqrt{n}}$ = standard error of the sample mean
s = sample standard deviation

Test Statistic

$$\text{Test statistic} = \frac{\text{Sample statistic - Hypothesized value}}{\text{Standard error of sample statistic}}$$

Power of a Test

Power of a test = 1 - P(Type II error)

Decision Rules for Hypothesis Tests

Decision	H_0 is True	H_0 is False
Do not reject H_0	Correct decision	Incorrect decision **Type II error**
Reject H_0	Incorrect decision **Type I error** Significance level = P(Type I error)	Correct decision Power of the test = 1 - P(Type II error)

Confidence Interval

$$\left[\left(\begin{matrix}\text{sample}\\\text{statistic}\end{matrix}\right) - \left(\begin{matrix}\text{critical}\\\text{value}\end{matrix}\right)\left(\begin{matrix}\text{standard}\\\text{error}\end{matrix}\right)\right] \leq \left(\begin{matrix}\text{population}\\\text{parameter}\end{matrix}\right) \leq \left[\left(\begin{matrix}\text{sample}\\\text{statistic}\end{matrix}\right) + \left(\begin{matrix}\text{critical}\\\text{value}\end{matrix}\right)\left(\begin{matrix}\text{standard}\\\text{error}\end{matrix}\right)\right]$$

$$\bar{x} - (z_{\alpha/2})\,(s/\sqrt{n}) \leq \mu_0 \leq \bar{x} + (z_{\alpha/2})\,(s/\sqrt{n})$$

Summary

Type of test	Null hypothesis	Alternate hypothesis	Reject null if	Fail to reject null if	P-value represents
One tailed (upper tail) test	$H_0: \mu \leq \mu_0$	$H_a: \mu > \mu_0$	Test statistic > critical value	Test statistic ≤ critical value	Probability that lies above the computed test statistic.
One tailed (lower tail) test	$H_0: \mu \geq \mu_0$	$H_a: \mu < \mu_0$	Test statistic < critical value	Test statistic ≥ critical value	Probability that lies below the computed test statistic.
Two-tailed	$H_0: \mu = \mu_0$	$H_a: \mu \neq \mu_0$	Test statistic < Lower critical value Test statistic > Upper critical value	Lower critical value ≤ test statistic ≤ Upper critical value	Probability that lies above the positive value of the computed test statistic *plus* the probability that lies below the negative value of the computed test statistic

t-Statistic

$$\text{t-stat} = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$$

Where:
x = sample mean
μ_0= hypothesized population mean
s = standard deviation of the sample
n = sample size

z-Statistic

$$\text{z-stat} = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}$$

Where:
x = sample mean
μ_0= hypothesized population mean
σ = standard deviation of the population
n = sample size

$$\text{z-stat} = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$$

Where:
x = sample mean
μ_0= hypothesized population mean
s = standard deviation of the sample
n = sample size

Tests for Means when Population Variances are Assumed Equal

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\left(\frac{s_p^2}{n_1} + \frac{s_p^2}{n_2}\right)^{1/2}}$$

Where:

$$s_p^2 = \frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1 + n_2 - 2}$$

s_1^2 = variance of the first sample

s_2^2 = variance of the second sample

n_1 = number of observations in first sample

n_2 = number of observations in second sample

degrees of freedom = $n_1 + n_2 - 2$

Tests for Means when Population Variances are Assumed Unequal

$$\text{t-stat} = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^{1/2}}$$

$$df = \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\frac{(s_1^2/n_1)^2}{n_1} + \frac{(s_2^2/n_2)^2}{n_2}}$$

Where:

s_1^2 = variance of the first sample

s_2^2 = variance of the second sample

n_1 = number of observations in first sample

n_2 = number of observations in second sample

Paired Comparisons Test

$$t = \frac{\bar{d} - \mu_{dz}}{s_{\bar{d}}}$$

Where:

$\bar{d}$ = sample mean difference

$s_{\bar{d}}$ = standard error of the mean difference $= \frac{s_d}{\sqrt{n}}$

s_d = sample standard deviation

n = the number of paired observations

Hypothesis Tests Concerning the Mean of Two Populations - Appropriate Tests

Population distribution	Relationship between samples	Assumption regarding variance	Type of test
Normal	Independent	Equal	t-test pooled variance
Normal	Independent	Unequal	t-test with variance not pooled
Normal	Dependent	N/A	t-test with paired comparisons

Chi Squared Test-Statistic

$$\chi^2 = \frac{(n-1)s^2}{\sigma_0^2}$$

Where:
n = sample size
s^2 = sample variance
σ_0^2 = hypothesized value for population variance

Test-Statistic for the F-Test

$$F = \frac{s_1^2}{s_2^2}$$

Where:
s_1^2 = Variance of sample drawn from Population 1
s_2^2 = Variance of sample drawn from Population 2

Hypothesis tests concerning the variance.

Hypothesis Test Concerning	Appropriate test statistic
Variance of a single, normally distributed population	Chi-square stat
Equality of variance of two independent, normally distributed populations	F-stat